6.4 THE RADIATION FIELD AND Q OF A RESONANT CYLINDRICAL PLASMA COLUMN

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A cylindrical plasma column placed across a waveguide or situated in free space in such a fashion that the electric field of an incoming wave is perpendicular to the axis of the column displays a series of resonant responses constituting a reflection and absorption spectrum.[1] The spectrum may be elicited and observed at a fixed microwave frequency by varying the current through the column on which the incoming wave impinges and noting the reflected power. The present paper discusses (1) the electromagnetic field associated with the assumed electronic motion for the principal member of the spectrum and (2) the effect of radiation damping on the Q for this member.

The motion of the electrons in a plasma column whose diameter is small compared to wave length and which is operating in its principal mode has been discussed by Herlofson.[2] In this mode, the electrons are assumed to move transversely to the axis of the column, and all electrons are assumed to move in time phase. In this paper, using Herlofson's description as a point of departure, an array of Hertzian dipole elements, each one consisting of a disc of radius a and of thickness dz, is considered distributed along a line which is the axis of the column, also taken as the z-axis. The procedure is classical. An expression for the vector potential in the region surrounding this source is developed from which are derived, in cylindrical coordinates, expressions for the components of the electric and magnetic field. The radiation field has the following components:

$$H_z = \frac{A}{2\sqrt{\rho\lambda}} \sin\phi \sin(\beta\rho - \omega t)$$

$$E_\phi = \eta H_z$$

where

$$A = \pi a^2 n e v_o$$

a is the radius of the column, n is electron density, e is electron charge and electrons are assumed to move with a velocity given by

$$V = V_o \cos \omega t$$

The radiation pattern is dipolar if one considers the field distribution in a plan perpendicular to the z-axis.

An expression for that part of the Q arising from radiation losses is developed by finding, at a cylindrical surface just enclosing the plasma column, the ratio between that part of the Poynting vector which represents reactive power flow and that part which represents radiated power. The expression for Q is

$$Q = \left\{ u\left[J_o(u) J_1(u) - Y_o(u) Y_1(u) \right] + Y_1^2(u) - J_1^2(u) \right\} \pi/2$$

where $u = \beta a$

If u is such that $\frac{\lambda}{a}$ is greater than 10, to a good approximation:

$$Q = (\lambda/a)^2 / 2\pi^3$$

The frequency for resonance of the principal mode is $f_p\sqrt{2}$ where f_p is plasma frequency and the factor $\sqrt{2}$ arises because the plasma is cylindrical in form.

When the column is placed in the open, the observed Q, for the discharge tubes used and for the principal resonance, lies in the range from 10 to 20. One important contribution to line width arises from the degradation of microwave power into heat. The second important contribution arises from radiation losses at the principal mode and this contribution becomes particularly important for small values of the ratio of wave length to diameter. A plot of Q vs λ/a is shown and experimental results are given. In an experimental situation the two contributions to line width often are comparable in magnitude.

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[1]A. Dattner "The Plasma Resonator," Ericsson Technics No. 2, Stockholm, pp. 310-350, (1957).

W. D. Hershberger "Absorption and Reflection Spectrum of a Plasma" Journal of Applied Physics, vol. 31, pp. 417-422, February 1960.

[2]N. Herlofson" Plasma Resonance in Ionospheric Irregularities," Arkiv Fysik, vol. 3, No. 15, pp. 247-297, (1951)